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INDUCTION STARTING OF DIRECT CURRENT MOTORS

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PREFACE

The work on this thesis was performed at the United States Naval Postgraduate School at Annapolis Maryland during the period July 1947 to June 1948.

The topic was suggested in a letter to the Postgraduate School concerning Theses, from the Chief of the Bureau of Ships, Navy Department, Washington, D.C.

The authors wish to acknowledge the invaluable aid extended at all times, during the preparation of this thesis, by Professor Charles V.O.Terwilliger, and Professor Allen E. Vivell, both of the Electrical Engineering Department of the Postgraduate School.

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OBJECT

The purpose of this thesis is to develop a theory of direct current motor starting using a starting reactor instead of the conventional starting resistance.

Consideration is given to the possibility of applying this device to some practical installation, and to the effect which the characteristics of the load will have upon the feasibility of using such a starting device.

GENERAL DISCUSSION

A discussion of present day direct current motor starting devices is felt to be in order before any effort is made to develop any new theory. Several types are on the market, and all depend essentially upon variation of the resistance of the motor armature circuit for the success of their operation.

When the armature of a motor is at rest, there is no back electro-motive force within the armature winding. Consequently, should a motor, whose armature is at rest, be connected directly to the supply lines, the initial rush of current through its armature would be limited only by the resistance and self-inductance of the armature and series field windings. This current may reach a value many times greater than the normal full load current of the machine and very serious sparking and flashing at the commutator would result, if the winding could survive such treatment.

As an example, a 220 volt shunt motor designed for a normal full load current of 100 amperes will have an armature circuit resistance in the order of 0.1 ohm. If the effect of self-inductance is neglected, the initial rush of current into the armature would be 2200 amperes, or 22 times the normal full load current for the machine. Of course, as soon as the armature begins to rotate, the back electromotive force developed in the armature windings would tend to reduce this current but under no circumstances could

this counter voltage be brought into existance rapidly enough to prevent irreparable damage to the machine.

Thus, for all except the smallest machines, some device must be interposed between the motor armature circuit and its power supply. One method of starting motors, and the only one in common use today, is by variation of the resistance of a variable resistance external to the armature and in series with it as indicated in Figure 1. This resistance is gradually removed from the circuit by moving contact C to the left as the motor speeds up. This method requires a starting resistance whose value is not less than twice the supply voltage divided by the rated full load current of the machine. The resistor must also be capable of continuous variation during the starting period.

When it is desired to start a motor that is permanently connected to its mechanical load, it is necessary that the motor develop a relatively high starting torque. In the case of the shunt motor, therefore, it is quite important that the starting rheostat be connected in series with the armature circuit only and that the shunt field be subject to full supply voltage throughout the starting period. Should the shunt field be inadvertantly connected so that the full voltage was not available, due to the drop in the starting resistor, the motor simply would not start. The torque in a motor is proportional to the product of armature current and field flux. If either of these is reduced below certain values for a given machine, it will fail to start.

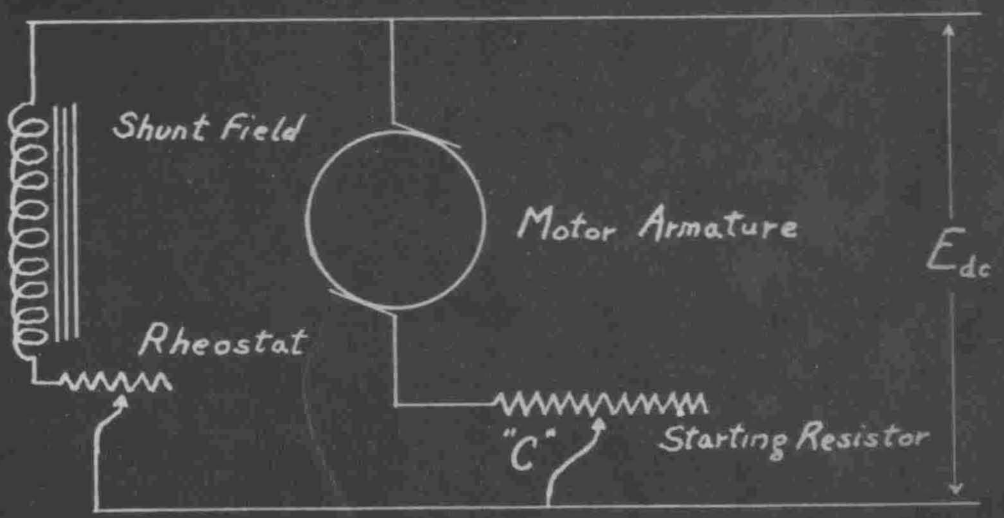
Shunt Field
Motor Armature
E_{dc}
Rheostat
"C"
Starting Resistor

FIGURE 1

In accelerating a motor from standstill to its rated speed, it is desirable (but not imperative) that the speed increase uniformly; that is, that the armature and its load have a constant acceleration. Since, for a given moment of inertia of the rotating masses, the angular acceleration is proportional to the net torque, it follows that the net torque must be constant if the angular acceleration is to be constant. The difference between total torque and the resisting torque, due to the load plus rotational losses in the motor itself, is the net torque. The resisting torques are so nearly constant that, in a particular application, it may be safely assumed that constant acceleration will be attained if a constant total torque is developed by the machine.

Since, as previously stated torque is proportional to the product of armature current and field flux, and field flux is to be at a constant and maximum value during the starting period, then armature current must be a constant during the starting period to have uniform acceleration of the rotating masses. (For any type motor, the field flux will be constant if line voltage is constant, and if armature current is constant, or very nearly so. However large variations of the armature current during starting will have some effect upon the field flux, particularly in a compound machine.)

Before any method of maintaining this armature current at a desired level is outlined, it is necessary to give

further consideration to the field flux and some of its effects upon the starting characteristics of the motor. If the field flux is fully set up prior to closing the armature circuit to the line, the sudden application of torque that results could easily damage the driven mechanism. In a series motor, where armature current and field flux are building up at the same time, the application of torque is most gradual and a kind of "cushioned" starting is obtained for the driven mechanism.

If the shunt field is so arranged that it closes just before, or at the same time as the armature circuit, a very similar cushioning effect is obtained for the shunt machine, due principally to the long delay in setting up the shunt field current. This delay is due to the transient characteristics of a circuit containing resistance and inductance. The shunt field in itself presents quite a large inductance in comparison to all the other parts of a direct current machine.

From the above, it is apparent that, from a purely mechanical viewpoint, and considering the fields alone, the smoothest kind of starting should be obtained in a compound motor in which the field switches close at the same time as the armature circuit. However, other considerations enter in a particular application, and there are many cases of shunt machines which are started with full field previously energized.

We will now consider the specific means used to arrive at a suitable resistive starting device and its effect upon armature current and motor speed during the starting period. It may be seen from the relation;

$$(1)\quad V = E_a + I_a R' = \frac{p\phi Z n}{60\times 10^8 a} + I_a R'$$

$$(2)\quad E_a = \phi\left[\frac{pZ}{60\times 10^8 a}\right] n = \phi Z' n = V - I_a R'$$

that if ϕ and I_a are both constant during the starting period, the resistance R' must be varied in such a way that, as "n" increases, the above relations are continuously satisfied. In other words;

$$(3)\quad R' = \frac{V}{I_a} - \frac{\phi Z'}{I_a} n$$

This means that R' will vary linearly and continously as a function of speed, subject to the assumption that armature current and ϕ Z' remain fixed. It is immediately evident that such continuous variation can be attained only in carbon pile rheostats or some liquid type rheostat. Neither of these is suitable for rugged or very prolonged use under the conditions in which the majority of direct current motors are operated.

The only type of resistor that is of sufficiently rugged construction to be fitted into all cases where such a starting method is to be used is one in which discreet units of resistance may be obtained by proper manual or automatic manipulation of a control device. This immediately forces

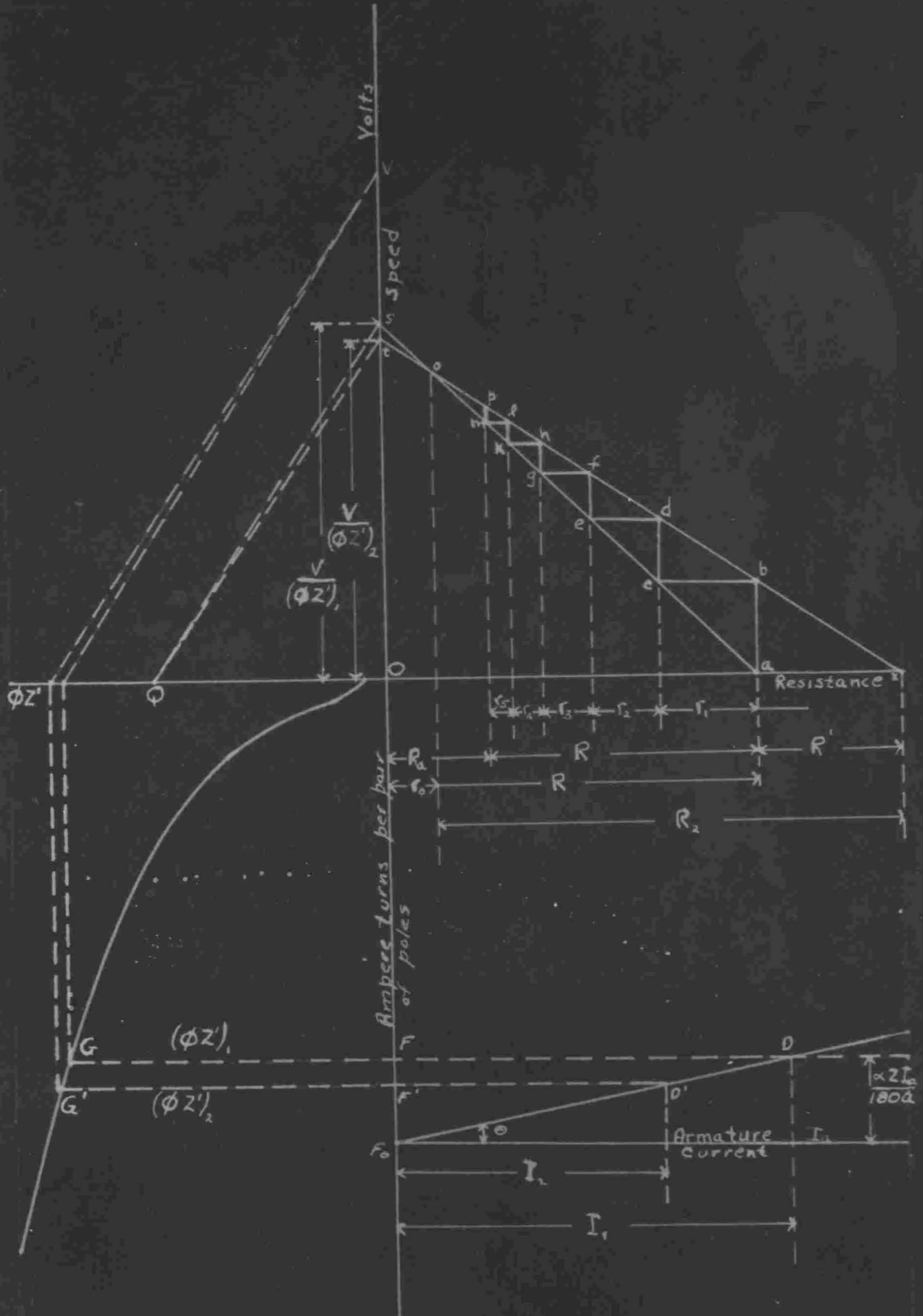
Volts
V
speed
S
t
o
p
l
m
n
K
g
f
d
e
c
b
a
Resistance
x
O
Q
ΦZ'
V/(ΦZ')₂
V/(ΦZ')₁
r₅
r₄
r₃
r₂
r₁
Rₐ
R
R'
r₀
R
R₂
Ampere turns per pair of poles
G
(ΦZ')₁
F
D
G'
(ΦZ')₂
F'
D'
θ
F₀
Armature current
I₂
I₁

Figure 2

an alteration of the original assumption that the armature current can remain constant during the starting period, ϕ since, with discreet steps of resistance, both current and Z' will vary in a step-by-step manner similar to the imposed step-by-step variation of resistance. The problem is immediately posed of how to proportion the steps of resistance so that the amount of variation of I_a and $Z'\Phi$ will remain within limits which will minimize the adverse effects upon torque and the acceleration characteristics of the machine.

Let us now find just how this resistance is proportioned in a shunt motor. Referring to Figure 1, let OG be the magnetization curve of the machine plotted as ampere turns per pair of poles on the vertical axis and $Z'\Phi$ along the horizontal axis. This curve is, of course, independent of the speed of the machine. Draw a line F_oD at an angle θ such that the intercept DR corresponding to OA (I_a) is equal to $\frac{\alpha Z I_a}{180 a}$, the demagnetizing ampere turns per pair of poles. The length OF_o represents the constant field of the machine. Let the starting resistance have a value R so that, at the moment of starting, the armature current is;

$$(4) \quad I_a = I_1 = \frac{V}{R_a + R}$$

If the current remained steady at this value, the relation

between the speed and the armature circuit resistance would be given;

$$(5) \quad n = \frac{V}{(\phi z')_1} - \frac{I_1 r}{(\phi z')_1}$$

which is represented by the straight line sa. The intercept on the speed axis (vertical) is $\frac{V}{(\phi z')_1}$ which corresponds to an r of zero. The intercept on the resistance axis (horizontal) is $R_a + R$, corresponding to a speed of zero.

This condition holds for only an instant, since the motor begins to turn over and build up a counter-voltage, which in turn reduces the current in the armature circuit. Assume now that the current has dropped to some new value different from the value with the motor armature at stand still. The line representing the relation between speed and armature circuit resistance would be dependent upon this new value of current. This lower current is the minimum to which we will allow armature current to drop before actually making a change in the resistance of the armature circuit. Such a line is represented by "tx" in Figure 1. It is now a simple operation to read off successive values of resistance which will maintain the armature current within a desired set of limits.

The condition to be satisfied at the end of all this manipulation of resistance is that when only R_a remains in the circuit the motor speed will have attained a value corresponding to the ordinate of point "p" and satisfying the relation;

(6) $n = \dfrac{V - I_2 R_a}{(\phi z')_2}$

Since the number of steps in the resistor is a whole number, it is clear that the choice of the current limits is not entirely arbitrary and that, in fact, a relation exists between them which is a function of the number of steps. However by the appropriate geometrical consideration of the diagram, the number of steps may be readily determined.

There are many types of starting rheostats, each designed for a particular kind of motor and for a particular application. The manually operated dial type rheostat was originally designed for all classes of service, but its use is now confined to motors not frequently started and where starting requirements are not severe. It has an inherent advantage in its low initial cost and cheap maintenance. Its great disadvantage is that its total operation is dependent upon the judgement of the operator. If the operator is in a hurry, severe damage to the motor may result, and if he is overcautious, the rheostat itself may easily be burned out. It does find specific application for motors up to 400 horsepower at 550 volts, with current as the limiting factor. This is due to the somewhat dangerous arching that occurs at the sliding contacts under high starting loads.

When service requirements call for frequent starts, stops, and reversals, the above type is replaced by the drum controller - also manually operated. This controller provides for starting the motor in either direction as would be required

by a crane hoist motor. This type of starter protects the operator from shock or burn, and the mechanism is shielded from dust, weather, and mechanical injury. Burning of the contacts is minimized by providing sufficient contact area and, except in the smallest sizes, by magnetic blowout coils (a coil connected in series with the current to be interrupted in order to set up a magnetic field for deflecting the electric arc). The drum controller does not in itself provide for protection against undervoltage or overload conditions. A circuit breaker must be provided if this protection is desired.

Manually controlled starters, while they can be arranged for automatic operation, are being replaced by fully automatic switching by means of controllers operated by electromagnets. This trend of practice developed because this arrangement allows remote control by means of push button stations with a minimum of auxiliary equipment. This type of control works independent of the operator and is essential in the cases of large motors driving such machinery as rolling mills, etc.

Several types of automatic controls have been developed. Counter E.M.F. control takes advantage of the fact that the E.M.F. increases as the speed of the motor increases - or the difference across the brushes will increase from zero at standstill to nearly line voltage at full speed. In general it involves connecting a coil of a magnetic contactor across the armature terminals and causing the contactor to close

when the armature voltage has reached some predetermined value determined by the air gap of the electromagnet. This then short circuits a part of the starting resistors. By providing a series of these contactors a means of cutting out the starting resistance step by step is provided. This scheme has the one disadvantage that the several contactors have different adjustments. However, this can be easily overcome by proper arrangement of the starting resistances and coils.

Series lockout control employs closing coils that respond to changes in the current being controlled. Closing coils may be put in series with the current being controlled, but a shunt type of closing coil may also be used employing an auxiliary contact opened or closed by a series relay. Control here is very similar to the counter E.M.F. arrangement.

In both the counter E.M.F. and series control the rate at which contactors short circuit successive sections of resistance depends upon the load being carried by the motor, since the load affects the acceleration, which affects the rate of building up of counter E.M.F. and consequent rate of decrease of initial current. With reference to the counter E.M.F. method variations in line voltage may introduce complications, since too high a voltage will close the contacts too soon, and, if it is too low, they may not close at all. In the series method if the speed rise is too rapid - with a light load - the variation in current may be too rapid to

be followed by the lockout device and the contactor may not close.

To correct these inherent disadvantages the time element control is designed to cut out successive sections of starting resistance at definite time intervals which are fixed by a timing device. One way of obtaining time acceleration is to drive the drum type controller by a small motor controlling time by adjusting the speed of the motor. A second method involves the mechanical closure of the successive contactors by means of a cam shaft driven by an auxiliary adjustable speed motor.

A fourth method combines the characteristics of time limit and series control and provides a time-current control. The time allowed on each step during acceleration is automatically varied in accordance with the load on the motor. This overcomes one disadvantage of the time limit control which cuts out the resistance at a fixed rate, regardless of the type of load. Time-current control allows more time per step, when the load is heavy, by reducing current peaks. Overload relays guard against the danger of a stalled motor and resultant overheating.

INDUCTION STARTING OF A DIRECT CURRENT MOTOR

Previous discussion has been limited to starting direct current motors with a starting resistance in series with the armature. The possibility of replacing that starting resistance with an inductance is now to be considered. It has been shown how the proper choice of resistance will give limiting values of maximum and minimum current during starting. Since the main problem in starting a direct current motor is to limit current and speed during starting so that unstable conditions will not result, the effects of replacing the starting resistance with inductance must be considered. Assuming conditions of constant load and therefore constant torque is the initial step. At this point it is further assumed that the current and speed will vary in some continuous manner - possibly as in Figure 3. It is felt that a limiting value of nearly one and a half times full load current during starting is a reasonable value. One of the problems that immediately develops, and one which may well limit the value of this starting method, is the manner of the variation of speed - especially its maximum value - with current, during the starting period.

Certain disadvantages are immediately evident. First, there exists no positive current limiting device should the motor fail to start, except that provided by the additional resistance of the inductor. It is felt, however, that this would be insufficient to protect the motor. Second, wide variation in load could conceivably influence the motor

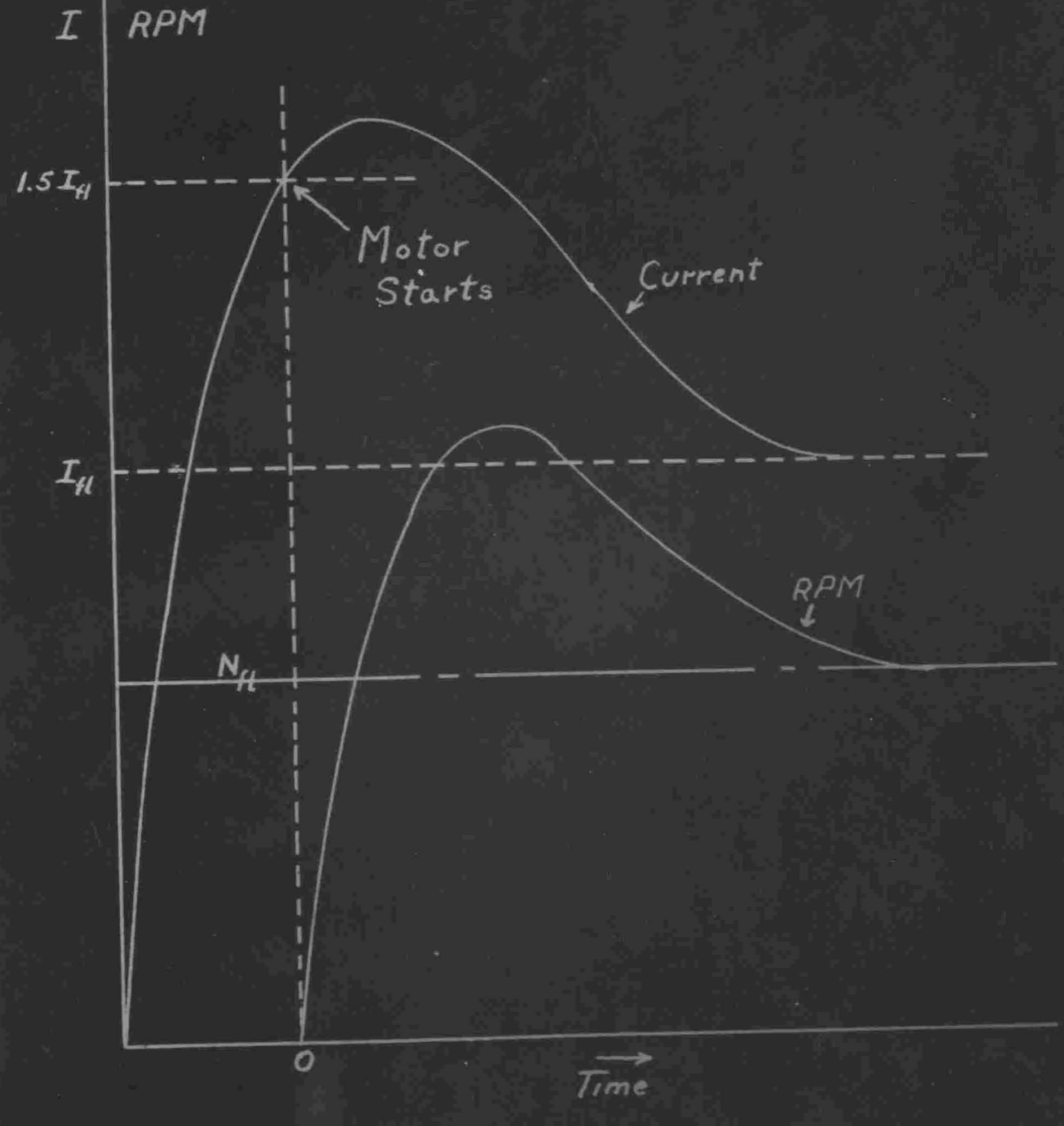

FIGURE 3

starting characteristics quite radically and adversely, if the variation occurred during the transient period.

Despite the above disadvantages it is hoped that certain advantages may be gained by use of such a starting device. First, the extreme simplicity of the starting mechanism since only one switch need be operated in the entire starting cycle. Second, a possibility of very smooth starting due to the continuous nature of current and speed curves as compared to the step by step characteristics of the resistance type starters. Third, the elimination of devices which might overheat if left in the circuit. In resistance type starters severe overheating often results if a short circuit switch across a resistor fails to close properly. Fourth, reasonable flexibility would be available for plugging operations. Finally, starting characteristics are independent of the operators judgment, or lack of it, during the starting period. Automatic timing devices could be applied to this mechanism.

MATHEMATICAL DEVELOPMENT

For the development of the analysis, the authors have chosen a simple machine, a direct - current shunt motor. At the outset, it was felt that an attempt to use a compound motor would lead to very complex mathematical problems which would be of no advantage in arriving at conclusions concerning the usefulness and the practicability of such a starting device. Figure 4 shows a diagram of the electrical circuit used in this analysis.

A voltage equation is written for the circuit as follows: (The symbol D represents $^{d}/dt$)

$$(1)\quad E_{dc} = (L+L_a)Di + (R+R_a)i + E_c$$

where Edc is line voltage

Ec is armature counter - voltage

$$(2)\quad E_c = \frac{p\phi Z n}{60\, a\, 10^8}$$

where p is the number of poles

a is the number of paths

ϕ is the flux per pole in maxwells

Z is the total number of armature conductors

n is the speed in revolutions per minute

The voltage equation is now written:

$$(1)\quad E_{dc} = (L+L_a)Di + (R+R_a)i + \frac{p\phi Z n}{60\, a\, 10^8}$$

Another equation showing an interrelation between n and i must now be obtained, since, during the starting

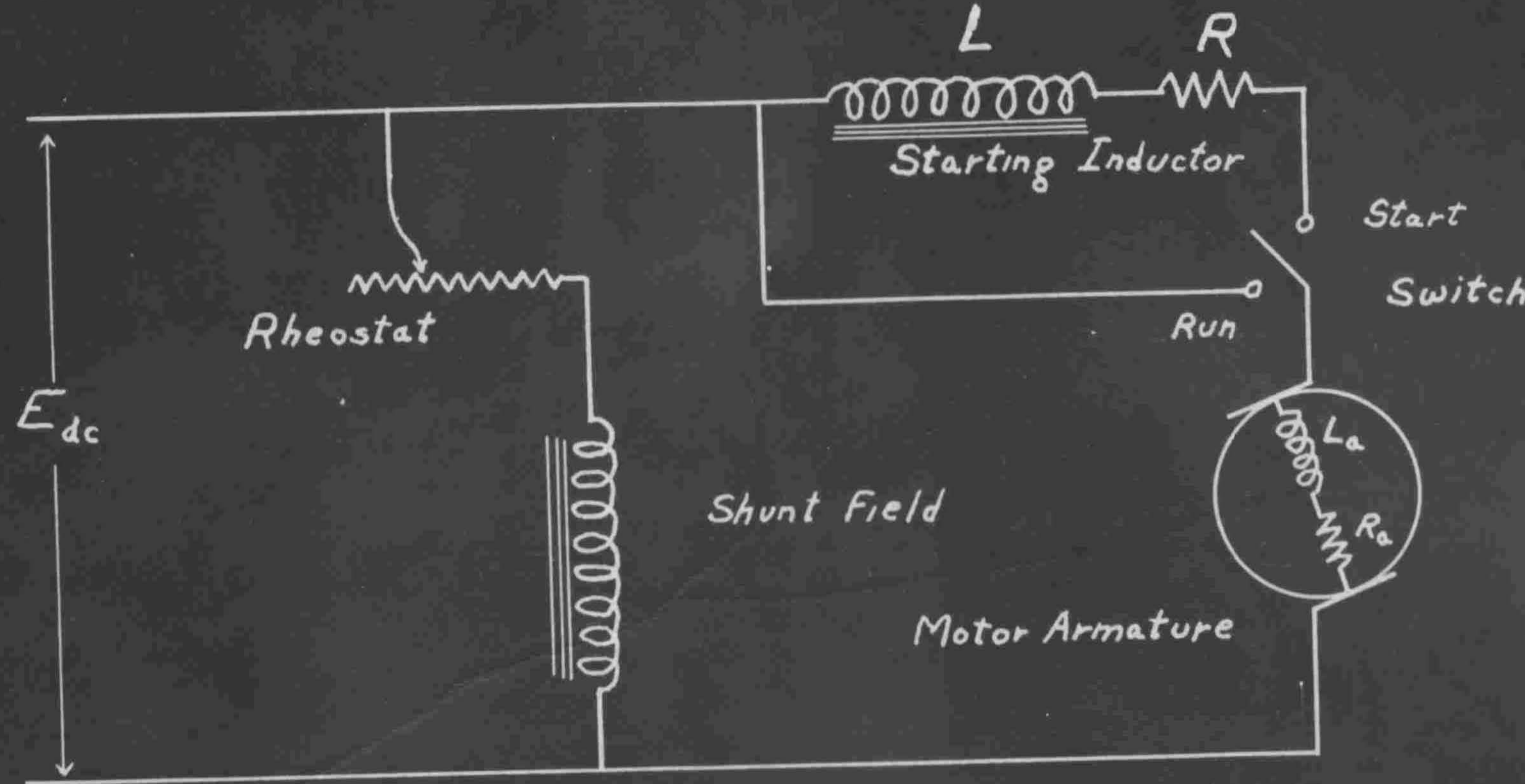
L
R
Starting Inductor
Start
Switch
Run
Rheostat
E_{dc}
Shunt Field
L_a
R_a
Motor Armature

FIGURE 4

period $\underline{n}$ varies from zero to rated speed, and in this starting device, will vary in some continuous manner instead of discreet steps as in resistive types of starters. If the equations for motor torque are used, such a relation is apparent.

$$(3) \quad T = T_a + T_f + T_L$$

T_a is the accelerating torque

T_f is the friction torque

T_L is the load torque

$\underline{T_f}$ and $\underline{T_L}$ are assumed to be constant during the starting period. This assumption is made to keep the analysis as simple as possible while making reasonable assumptions. It is realized that a direct current motor in use seldom has such a constant load imposed, but a load analysis is felt to be beyond the object of this thesis.

$$(4) \quad T = \frac{60 E_c i}{2\pi n} = \frac{p \phi Z i}{2\pi a \, 10^8}$$

This formula is converted to foot pound units by a constant to yield.

$$(4) \quad T = \frac{p \phi Z i}{8.511 \times 10^8 \, a}$$

(5) $$T_a = \frac{2\pi J}{60\,g} Dn$$

where J is the polar moment of inertia of the rotating parts of the motor and its load, g is the gravitational constant.

This formula is converted to foot pound units if J is in foot-pound-second-second and g is in feet per second per second to yield.

(5) $$T_a = \frac{J}{265.046} Dn$$

Let:

$$\frac{J}{265.046} = K_1 \qquad\qquad T_f + T_L = T_C$$

$$\frac{p\phi Z}{8.511 \times 10^8 a} = K_2$$

$$\frac{p\phi Z}{60 \times 10^8\, a} = K_3$$

Then:

(3) $$K_1 Dn = K_2 i - T_c$$

$$i = \frac{K_1}{K_2} Dn + \frac{T_c}{K_2}$$

$$Di = \frac{K_1}{K_2} D^2 n$$

Now substituting in equation (1)

$$(6)\quad \frac{K_1}{K_2}(L+L_a)\mathcal{D}^2 n + \frac{K_1}{K_2}(R+R_a)\mathcal{D}n + \frac{(R+R_a)}{K_2}T_c + K_3 n = E_{dc}$$

Let:

$$\frac{R+R_a}{L+L_a} = A \qquad \frac{K_2 E_{dc} - (R+R_a)T_c}{K_1(L+L_a)} = C$$

$$\frac{K_3 K_2}{K_1(L+L_a)} = B$$

$$(6)\quad \mathcal{D}^2 n + A\mathcal{D}n + Bn = C$$

$$(7)\quad n = \Theta\epsilon^{m_1 t} + \Psi\epsilon^{m_2 t} + \frac{C}{m_1 m_2}$$

$$\mathcal{D}n = m_1\Theta\epsilon^{m_1 t} + m_2\Psi\epsilon^{m_2 t}$$

Substituting in equation (3)

$$(8)\quad i = \frac{K_1}{K_2}m_1\Theta\epsilon^{m_1 t} + \frac{K_1}{K_2}m_2\Psi\epsilon^{m_2 t} + \frac{T_c}{K_2}$$

The known boundary conditions may be applied to these simultaneous equations to determine Θ and Ψ.

at $t = 0$

$i = 1.5I \qquad n = 0$

where I is the ultimate armature current after motor is running with load. An excess of fifty percent current is felt to be a reasonable value at which the static friction forces will be overcome and the motor will begin to turn.

$$\frac{K_1}{K_2}m_1\Theta + \frac{K_1}{K_2}m_2\Psi = 1.5I - \frac{T_c}{K_2}$$

$$\Theta + \Psi = -\frac{C}{m_1 m_2}$$

Solving simultaneously:

$$\Theta = \frac{1.5 I K_2}{K_1(m_1-m_2)} - \frac{T_c}{K_1(m_1-m_2)} + \frac{C}{m_1(m_1-m_2)}$$

$$\Psi = -\frac{1.5 I K_2}{K_1(m_1-m_2)} + \frac{T_c}{K_1(m_1-m_2)} - \frac{C}{m_2(m_1-m_2)}$$

An equation for both i and n may now be written in terms of circuit constants and t when it is noted that:

$$\frac{T_c}{K_2} = \frac{T_c}{p\phi Z} \, 8.511 \times 10^8 a = I$$

$$\frac{K_1 C}{K_2} = \frac{E_{dc} - (R+R_a) I}{L + L_a}$$

Substituting in equation (8)

$$i = \left[\frac{1.5 m_1 I}{(m_1-m_2)} - \frac{m_1 T_c}{K_2(m_1-m_2)} + \frac{K_1 C}{K_2(m_1-m_2)}\right] \epsilon^{m_1 t}$$

$$- \left[\frac{1.5 m_2 I}{(m_1-m_2)} - \frac{m_2 T_c}{K_2(m_1-m_2)} + \frac{K_1 C}{K_2(m_1-m_2)}\right] \epsilon^{m_2 t} + \frac{T_c}{K_2}$$

This becomes:

$$(9) \quad i = \left[.5 I \frac{m_1}{m_1-m_2} + \frac{E_{dc} - (R+R_a) I}{L+L_a} \times \frac{1}{m_1-m_2}\right] \epsilon^{m_1 t}$$

$$- \left[.5 I \frac{m_2}{m_1-m_2} + \frac{E_{dc} - (R+R_a) I}{L+L_a} \times \frac{1}{m_1-m_2}\right] \epsilon^{m_2 t} + I$$

In equation (9):

$$m_1 = -\frac{A}{2} + \frac{1}{2}\sqrt{A^2 - 4B}$$

$$m_2 = -\frac{A}{2} - \frac{1}{2}\sqrt{A^2 - 4B}$$

$$A = \frac{R+R_a}{L+L_a}$$

$$\sqrt{A^2-4B} = \sqrt{\left(\frac{R+R_a}{L+L_a}\right)^2 - \frac{p^2\,\phi^2\,Z^2}{0.4817 \times 10^{16}\, a^2\, J\, (L+L_a)}}$$

It is apparent that, if M_1 and M_2 are real, equation (9) will yield the curve of current versus time for any given inductance. If, as is more generally the case, M_1 and M_2 are complex numbers, a further revision of equation (9) is necessary to obtain such a curve.

Let:

$$m_1 = -\alpha + j\beta$$

$$m_2 = -\alpha - j\beta$$

$$\alpha = \frac{A}{2}$$

$$\beta = \frac{1}{2}\sqrt{4B - A^2}$$

$$m_1 - m_2 = 2j\beta$$

$$W = .5I$$

$$Y = \frac{E_{dc} - (R+R_a)I}{L+L_a}$$

Equation (9) is now written:

$$(9) \quad i = \left[W \frac{-\alpha+j\beta}{2j\beta} + \frac{Y}{2j\beta}\right] \epsilon^{(-\alpha+j\beta)t}$$

$$- \left[W \frac{-\alpha-j\beta}{2j\beta} + \frac{Y}{2j\beta}\right] \epsilon^{(-\alpha-j\beta)t} + I$$

$$i = \epsilon^{-\alpha t}\left[\frac{-W\alpha\epsilon^{j\beta t} + W\alpha\epsilon^{-j\beta t} + Y\epsilon^{j\beta t} - Y\epsilon^{-j\beta t}}{2j\beta}\right]$$

$$+ \epsilon^{-\alpha t}\left[\frac{jW\beta\epsilon^{j\beta t} + jW\beta\epsilon^{-j\beta t}}{2j\beta}\right] + I$$

$$\frac{\epsilon^{j\beta t} - \epsilon^{-j\beta t}}{2j} = \sin\beta t$$

$$\frac{\epsilon^{j\beta t} + \epsilon^{-j\beta t}}{2} = \cos\beta t$$

$$i = \left[\frac{Y - W\alpha}{\beta} \sin\beta t + W\cos\beta t\right]\epsilon^{-\alpha t} + I$$

This yields equation (10)

$$(10) \quad i = \left[\frac{\frac{E_{dc} - (R+R_a)I}{L+L_a} - .5 I\alpha}{\beta} \sin\beta t +\right.$$

$$\left. .5 I \cos\beta t\right]\epsilon^{-\alpha t} + I$$

This is the second solution for the initial differential equation and is the most useful form for inductances of large enough value to successfully start the usual direct current motor. A determination of the two solutions for the motor speed is not necessary since only the oscillatory case will be of value in any calculations.

Using equation (7), a direct solution may be obtained where:

M_1 and M_2 are the same as before

$$F = \frac{.5 I K_2}{K_1}$$

C is as defined on page 21

$$(7) \quad n = \frac{F}{2j\beta}\epsilon^{(-\alpha+j\beta)t} - \frac{F}{2j\beta}\epsilon^{(-\alpha-j\beta)t}$$

$$+ \frac{C(-\alpha-j\beta)}{(\alpha^2+\beta^2)2j\beta}\epsilon^{(-\alpha+j\beta)t} - \frac{C(-\alpha+j\beta)}{(\alpha^2+\beta^2)2j\beta}\epsilon^{(-\alpha-j\beta)t}$$

$$+ \frac{C}{\alpha^2+\beta^2}$$

Simplifying and collecting terms

$$n = \left[\left(\frac{F}{\beta} - \frac{C\alpha}{(\alpha^2+\beta^2)\beta}\right)\sin\beta t - \frac{C}{\alpha^2+\beta^2}\cos\beta t\right]\epsilon^{-\alpha t}$$

$$+ \frac{C}{\alpha^2+\beta^2}$$

Then:

$$(11)\quad n = \frac{31.142\,p\phi Z}{10^8\ aJ}\left\{\left[\left(\frac{.5I}{\beta} - \frac{\alpha}{\alpha^2+\beta^2}\times\frac{E_{dc}-(R+R_a)I}{\beta\ (L+L_a)}\right)\sin\beta t\right.\right.$$

$$\left.\left. - \frac{E_{dc}-(R+R_a)I}{(\alpha^2+\beta^2)(L+L_a)}\cos\beta t\right]\epsilon^{-\alpha t} + \frac{E_{dc}-(R+R_a)I}{(\alpha^2+\beta^2)(L+L_a)}\right\}$$

Calculations using equations (10) and (11) were made and are indicated in the appendix. A set of curves for a particular value of inductance is shown in Figure 5.

It is apparent that, though the armature current reaches full value for a given load, the motor speed will never be full speed for a given current until the extra inductance of the starting device and its resistance are removed from the armature circuit. This "cut-out" point should be selected so that a minimum transient will occur at the time full line voltage is applied directly to the terminals of the motor. For this reason, a further application of the original differential equations with altered boundary conditions is necessary. An essentially parallel method of solution is employed as follows:

$$t = 0$$
$$i = I'$$
$$n = N'$$

Applying these boundary conditions to equations (7) and (8) and solving

$$\Theta = \frac{I' K_2}{K_1\,(m_1-m_2)} - \frac{T_c}{K_1\,(m_1-m_2)} - \frac{m_2 N'}{m_1-m_2} + \frac{C}{m_1\,(m_1-m_2)}$$

$$\psi = -\frac{I'K_2}{K_1(m_1-m_2)} + \frac{T_c}{K_1(m_1-m_2)} + \frac{m_1 N'}{m_1 - m_2} - \frac{C}{m_1(m_1-m_2)}$$

Using the same approach as before, two solutions for n and i may be found. Only the oscillatory case will be given since the characteristics of any direct current motor are such as to make only this solution useful. In these equations:

$$\alpha = \frac{R_a}{2L_a}$$

$$\beta = \frac{1}{2}\sqrt{\frac{p^2\phi^2 Z^2}{0.4817\times 10^{16}\, a^2 J L_a} - \left(\frac{R_a}{L_a}\right)^2}$$

(12)
$$i = \left[\frac{\dfrac{E_{dc} - R_a I}{L_a} - \dfrac{N' a J(\alpha^2+\beta^2)10^8}{31.142\, p\phi Z} - (I'-I)\alpha}{\beta}\sin\beta t + (I'-I)\cos\beta t\right]\epsilon^{-\alpha t} + I$$

(13)
$$n = \left[\frac{\dfrac{(I' - \frac{T_c}{K_2})K_2}{K_1} + N'\alpha - \dfrac{C\alpha}{\alpha^2+\beta^2}}{\beta}\sin\beta t + \left(N' - \frac{C}{\alpha^2+\beta^2}\right)\cos\beta t\right]\epsilon^{-\alpha t} + \frac{C}{\alpha^2+\beta^2}$$

Values of K_2, K_1 and C were not substituted in equation (13) since it is possible to use the equations without such substitution.

CONCLUSIONS

As a result of the mathematical calculations and data submitted in the Appendix, the following conclusions have been reached;

(1) The motor will hunt severely during the starting period.

(2) Armature current is easily limited by the use of an inductance, but a large oscillation of the current is introduced, before a constant value is attained.

(3) The resistance of the starting inductor must be carefully considered before any such device is designed, because of the most marked effect of resistance upon the starting characteristics.

(4) The size of such a starting device is large both in comparison to the physical dimensions of the motor it is to serve, and, in addition, to the physical dimensions of an ordinary resistance type starter that will successfully operate the motor under most conditions.

In reviewing the facts derived from consideration of the calculations and the curves of starting characteristics, it does not appear that any improvement may be made to the tendency of the machine to hunt while starting. This phenomenon is inherent in the derivation of the curves that show the starting characteristics.

This device does answer the basic requirements of a starting mechanism in that current in the armature circuit is limited while the motor is being brought up to speed. The oscillation of current is not, of itself, a disadvantage, if some way to stop the possibly destructive speed oscillations were available. Of course, current

and speed are related and oscillation in one will either produce, or be brought about by, the oscillatory characteristics of the other.

In general, the results indicate that a direct-current shunt motor may be started by means of an inductive starting device in series with the armature circuit, but that, during the starting period, what may be serious, and, certainly objectionable hunting in the speed of the motor will be inherently present. Such a starting device is simple, provides for elimination of heating and ultimate destruction of the starting device in event of maloperation (as may occur in a resistive starter), provides a reasonable flexibility, and eliminates the errors of judgement which are the usual cause for failure in operation of present day starting devices.

It should also be pointed out that further work remains to be carried out along this line. The possibility of a variation of field strength during starting having an effect upon the characteristics should definitely be considered. This would alter the mathematics of the problem, and could possibly lead to more satisfactory results. Thus, the device should be useful for series and compound motors where large field variation occurs during the starting period. The overall effect remains, however, quite unpredictable, and would require extensive mathematical analysis, which is felt to be beyond the scope of this particular report and the time allotted for its completion.

Similarly, variation of torque during the starting period may have a very marked effect, since the foregoing work assumed a constant torque. Consideration of such a variation would require another lengthy treatise including a torque analysis for each particular application of the motor.

Mutual inductance between the motor armature and the field has been assumed negligible in this thesis. Further work along this line, particularly in the series and compound machines, might prove fruitful in the elimination of the hunting found in the shunt machine. There is also the effect of mutual inductance between the series and shunt fields that poses a very interesting mathematical problem in conjunction with a necessity for extensive laboratory research into field characteristics in general, during the starting period.

The ultimate conclusion of this thesis is that an inductive type starting device can be successfully used on a direct current shunt motor under the conditions assumed, but that such a device is impractical of application in most situations due to hunting during starting and the prohibitive size of the inductor.

APPENDIX

Calculations were made on a starting device for a motor in the Electrical Engineering laboratory at the U. S. Naval Postgraduate School. This motor had the following constants:

Number of poles -	p	4
Number of paths -	a	2
Total flux per pole -	ϕ	800,000 maxwells
Number of armature conductors -	Z	444
Armature resistance -	Ra	.119 ohms
Armature inductance -	La	.00486 henries
Full load current -	I_{fl}	60 amperes
Total moment of inertia -	J	14.133 Lb.ft.sec.^{2}
Line voltage -	Edc	115 volts

Assumed resistance of starting inductor -	R	.5 ohms
Assumed inductance of starting inductor -	L	5 Henries

All motor constants were obtained from the design data loaned by the Westinghouse Electric and Manufacturing Company except for La and J.

Determination of La:

An empirical formula was used for this constant as follows:

$$L_a = \frac{100\, E_{dc}}{\left(\frac{RPM}{100}\right)^2 \left(\frac{p}{a}\right) p\, I_{fl}\, D\, l}$$

where:

RPM	armature rated speed
D	armature diameter in inches
l	armature core length in inches

Determination of J:

The moment of inertia of the armatures of the motor, its attached generator rotor, and its couplings was determined experimentally. A rack to support the armature was manufactured in the machine shop at the Postgraduate School. This rack supported the motor so that it could be rolled along an arc of twelve inch radius. The oscillation of the armature assembly was timed by stop-watch and the moment of inertia was determined by the formula:

$$I = \frac{W r^2 \left[T^2 - \frac{R-r}{g}\right]}{R-r}$$

where:

264	W - weight of armature assembly in pounds
.07292	r - radius of armature shaft in feet
3.06 secs.	T - time of oscillation during one cycle
32.174	g - gravitational constant
1	R - radius of path of armature oscillation.

The formulas were placed in a simplified form for ease of calculation as follows:

$$i = \epsilon^{-\alpha t}\left[\frac{\frac{E_{dc} - (R+R_a)I_{f1}}{L+L_a} - .5 I_{f1}\alpha}{\beta} \sin \beta t + .5 I_{f1} \cos \beta t\right] + I_{f1}$$

$$\alpha = \frac{R+R_a}{2(L+L_a)} \qquad \beta = \sqrt{\frac{p^2 \phi^2 Z^2}{0.4817 \times 10^{16}\, a^2 J\, (L+L_a)} - \left(\frac{R+R_a}{L+L_a}\right)^2}$$

Substitution of values yields:

$$\alpha = 0.0619$$

$$\beta = 0.605$$

$$A = \frac{\frac{E_{dc} - (R + R_a) I_{f1}}{L + L_a} - .5 I_{f1} \alpha}{\beta}$$

$$B = \frac{1}{2} I_{f1}$$

$$A = 22.421$$

$$B = 30$$

The current equation is now written:

$$I = \epsilon^{-\alpha t}\left[A \sin \beta t + B \cos \beta t\right] + I_{f1}$$

The equation for motor speed:

$$n = \epsilon^{-\alpha t}\left\{\left[\frac{15.571\ p \phi Z I_{f1}}{a J \beta\ 10^{8}} - \frac{31.142\ p \phi Z \alpha\ 10^{-8}}{\beta (\alpha^2 + \beta^2) a J (L + L_a)} \left(E_{dc} - \{R + R_a\} I_{f1}\right)\right] \right.$$

$$\left. \sin \beta t - \left[\frac{31.142\ p \phi Z\ 10^{-8}}{(\alpha^2 + \beta^2) a J\ (L + L_a)} \left(E_{dc} - \{R + R_a\} I_{f1}\right)\right] \cos \beta t \right\} +$$

$$\frac{31.142\ p \phi Z\ 10^{-8}}{(\alpha^2 + \beta^2)\ a J\ (L + L_a)} \left(E_{dc} - \{R + R_a\} I_{f1}\right)$$

This expression is quite cumbersome and may be rewritten for simplicity:

$$n = \epsilon^{-\alpha t}\left[E \sin \beta t - F \cos \beta t\right] + G$$

where:

$$\alpha = 0.0619$$

$$\beta = 0.605$$

and

$$E = \frac{15.571\, p \phi Z I_{f1}}{a J \beta\, 10^{8}} - \frac{31.142\, p \phi Z \alpha\, 10^{-8}}{\beta(\alpha^2+\beta^2)\, a J\, (L+L_a)} \left(E_{dc} - (R+R_a) I_{f1}\right)$$

$$F = \frac{31.142\, p \phi Z\, 10^{-8}}{(\alpha^2+\beta^2)\, a J\, [L+L_a]}\left[E_{dc} - (R+R_a) I_{f1}\right]$$

$$F = G$$

Substitution of values gives

$$E = 708.84$$

$$F = 658.43$$

Data sheets are appended with a plot of speed versus time and current versus time.

t	αt	βt	$B\cos\beta t$	$A\sin\beta t$	I	$E\sin\beta t$	$F\cos\beta t$	R.P.M.
.1	.00619	.0605	29.78	1.352	90.82	42.7	656	55
.2	.01238	.121	29.80	2.71	92.1	85.5	653	97
.3	.01858	.1815	29.50	4.04	92.9	127.8	645	150
.4	.02479	.242	29.38	5.37	93.9	168.2	639	198
.5	.03095	.3025	28.6	6.67	94.2	209.5	626	254
.6	.03718	.363	28.00	7.86	94.5	251	614	309
.7	.04338	.424	27.35	9.22	95	291	600	362
.8	.04960	.484	26.60	10.48	95.45	330	594	407
.9	.05575	.545	25.65	11.61	95.3	366	563	471.5
1.0	.0619	.605	24.65	12.73	95.1	402	541	527.5
1.1	.0681	.666	23.60	13.85	95	437	517	546
1.2	.0743	.726	22.45	14.90	94.7	469	491	637.6
1.3	.0805	.787	21.20	15.89	94.2	500	465	690.3
1.4	.0867	.847	19.88	16.80	93.65	530	435	745.1
1.5	.0929	.907	18.49	17.65	93	557	406	786.5
1.6	.0991	.968	17.05	18.49	92.2	583	372	849
1.7	.1054	1.03	15.43	19.22	91.2	607	338	899
1.8	.1116	1.09	13.89	19.90	90.2	627	304	937
1.9	.1178	1.151	12.23	20.5	89.15	645	268	993
2.0	.1238	1.211	10.6	21	87.9	662	232	1032
2.1	.1302	1.271	8.85	21.45	86.65	676	194	1080
2.2	.1364	1.332	7.1	21.8	85.25	687	156	1122
2.3	.1425	1.394	5.26	22.1	83.7	697	115.5	1154
2.4	.1489	1.455	3.46	22.3	82.2	702	75.9	1199
2.5	.1551	1.515	1.67	22.4	80.65	706	36.7	1230
2.6	.1612	1.575	- .126	22.42	78.99	708	- 2.76	1263

t	αt	βt	$B\cos\beta t$	$A\sin\beta t$	I	$E\sin\beta t$	$F\cos\beta t$	R.P.M.
2.7	.1673	1.636	- 1.96	22.38	77.25	705	- 42.9	1290
2.8	.1737	1.697	- 3.78	22.28	75.55	702	- 82.8	1317
2.9	.1799	1.757	- 5.56	22.05	73.76	695	-122	1340
3.0	.1857	1.815	- 7.25	21.8	72.1	686	-159	1360
3.2	.198	1.936	-10.71	20.9	68.35	662	-235	1395
3.4	.2105	2.06	-14.1	19.8	64.62	625	-308	1414
3.6	.2227	2.18	-17.2	18.39	60.95	582	-376	1424
3.8	.235	2.30	-20.00	16.75	57.44	528	-438	1422
4.0	.2465	2.42	-22.58	14.8	53.94	468	-493	1408
4.2	.260	2.54	-24.7	12.7	50.76	401	-541	1385
4.4	.2725	2.661	-26.6	10.4	47.68	328	-583	1352
4.6	.2847	2.785	-28.1	7.84	44.8	247	-616	1307
4.8	.297	2.91	-29.2	5.14	42.15	162.5	-640	1254
5.0	.3095	3.025	-29.8	2.60	40.05	82.5	-654	1205
5.2	.322	3.15	-29.99	- .188	38.15	- 5.95	-658	1130
5.4	.3345	3.27	-29.8	-2.87	36.6	-90.8	-652	1060
5.6	.347	3.39	-29.05	-5.52	35.6	-174	-637	985
5.8	.359	3.51	-28.0	-8.07	34.8	-255	-613	908
6.0	.371	3.63	-26.5	-10.55	34.4	-332	-580	830
6.2	.384	3.75	-24.6	-12.88	34.5	-405	-540	750
6.4	.396	3.88	-22.15	-15.14	34.9	-477	-486	664
6.6	.408	3.99	-19.85	-16.85	35.6	-532	-435	593
6.8	.421	4.12	-16.75	-18.61	36.75	-587	-367	514
7.0	.434	4.29	-13.65	-20.0	38.25	-631	-299	443
7.2	.446	4.36	-10.36	-21.1	39.8	-665	-227	377
7.4	.458	4.48	- 6.9	-21.81	41.85	-689	-151.5	318

t	αt	βt	$B\cos\beta t$	$A\sin\beta t$	I	$E\sin\beta t$	$F\cos\beta t$	R.P.M.
7.6	.471	4.60	- 3.37	-22.3	44	-703	-73.7	265
7.8	.483	4.72	+ .228	-22.42	46.3	-707	+ 5.0	220
8.0	.496	4.84	+ 3.81	-22.3	48.72	-703	+83.6	180
8.2	.507	4.96	+ 7.35	-21.67	51.36	-686	161	146
8.4	.52	5.08	10.78	-20.95	53.95	-661	236	125
8.6	.532	5.21	14.33	-19.72	56.83	-622	315	107
8.8	.544	5.32	17.15	-18.45	59.24	-582	376	101
9.0	.557	5.45	20.2	-16.62	62.05	-525	442	98
9.2	.569	5.56	22.5	-14.85	64.33	-468	493	115
9.4	.581	5.68	24.7	-12.7	66.71	-402	542	130
9.6	.594	5.81	26.7	-10.25	69.08	-324	587	155
9.8	.606	5.92	28.0	- 7.95	70.95	-251	616	175
10.0	.619	6.05	29.2	- 5.18	72.92	-164	641	224
10.2	.621	6.17	29.8	- 2.53	74.65	- 80	655	263
10.4	.634	6.29	29.99	.153	75.98	+ 4.8	658	311
10.6	.646	6.42	29.75	3.06	77.2	+ 96.5	653	367
10.8	.657	6.54	29	5.19	77.7	+179	638	420
11	.67	6.65	27	8.65	78.25	254	615	473
11.2	.683	6.77	26.5	10.5	78.65	331	582	532
11.4	.694	6.90	24.5	12.95	78.68	409	538	594
11.6	.707	7.02	22.25	15.1	78.4	475	488	652
11.8	.719	7.14	19.62	17	77.85	535	431	709
12	.731	7.26	16.8	18.6	77.15	586	369	763
12.2	.744	7.38	13.7	19.91	76.0	629	301	814
12.4	.755	7.5	10.4	21.1	74.8	664	228	863
12.6	.766	7.62	6.95	21.8	73.32	688	152	907

t	α t	β t	Bcosβt	Asinβt	I	Esinβt	Fcosβt	R.P.M.
12.8	.78	7.74	3.41	22.25	71.25	702	75	945
13.0	.792	7.86	- .18	22.42	70.08	708	- 3.9	981
13.2	.804	7.98	-3.76	22.22	68.26	701	-83	1009
13.4	.810	8.1	-7.3	21.75	66.42	685	-161	1034
13.6	.829	8.23	-11	20.85	64.3	658	-239	1050
13.8	.841	8.35	-14.3	19.7	62.33	618	-314	1061
14	.852	8.47	-17.3	18.3	60.43	577	-381	1067
14.2	.865	8.59	-20.18	16.6	58.49	525	-443	1066
14.4	.877	8.7	-22.4	14.88	56.88	468	-493	1057
14.6	.889	8.84	-25	12.4	54.83	392	-549	1044
14.8	.902	8.95	-26.65	10.28	53.36	324	-589	1029
15	.914	9.06	-28	8.0	52.00	253	-615	1005
15.2	.940	9.19	-29.2	5.21	50.64	165	-641	973
15.4	.953	9.32	-29.85	2.34	49.4	74	-654	939
15.6	.966	9.43	-29.99	- .116	48.53	- 3.68	-657	906
15.8	.979	9.57	-29.65	-3.24	47.66	-102	-651	864
16.0	.990	9.67	-29.15	-5.44	47.17	-172	-639	832
16.2	1.002	9.80	-27.95	-8.22	46.73	-259	-615	789
16.4	1.015	9.92	-26.40	-10.68	46.59	-336	-579	746
16.6	1.026	10.05	-24.35	-13.15	46.58	-415	-534	701
16.8	1.040	10.16	-22.15	-15.05	46.97	-475	-489	663
17.0	1.052	10.29	-19.45	-17.10	47.25	-539	-427	620
17.2	1.064	10.41	-16.59	-18.65	47.83	-590	-364	580
17.4	1.077	10.52	-13.72	-19.91	48.52	-636	-302	544
17.6	1.089	10.65	-10.16	-21.1	49.46	-666	-223	509
17.8	1.102	10.78	- 6.11	-21.9	50.53	-691	-141	475

t	αt	βt	$B\cos\beta t$	$A\sin\beta t$	I	$E\sin\beta t$	$F\cos\beta t$	R.P.M.
18.0	1.113	10.89	− 3.16	−22.3	51.64	−704	− 69.5	449
18.2	1.126	11.02	+ .732	−22.4	52.96	−706	16.1	424
18.4	1.139	11.14	+ 4.31	−22.1	54.30	−700	94.6	404
18.6	1.151	11.26	7.85	−21.62	55.64	−683	173	387
18.8	1.162	11.38	11.25	−20.80	57.32	−656	246	376
19.0	1.175	11.50	14.50	−19.65	58.41	−620	318	370
19.2	1.188	11.61	17.30	−18.34	59.68	−578	379	366
19.4	1.201	11.74	20.32	−16.50	61.14	−521	445	367
19.6	1.213	11.86	22.85	−14.55	62.47	−459	501	373
19.8	1.225	11.99	25.15	−12.21	63.80	−386	551	383
20.0	1.233	12.12	27.10	− 9.68	65.05	−306	584	400
20.2	1.251	12.23	28.30	− 7.41	65.97	−234	621	413
20.4	1.263	12.35	29.30	− 4.82	66.93	−152	642	433
20.6	1.275	12.47	29.90	− 2.16	67.73	− 68	655	456
20.8	1.288	12.60	29.98	+ .754	68.46	23.8	657	484
21.0	1.300	12.72	29.65	+ 3.43	69.02	108	650	510
21.2	1.312	12.83	29.00	+ 5.85	69.40	184	635	537
21.4	1.325	12.96	27.70	+ 8.60	69.65	271	607	572
21.6	1.333	13.08	26.15	11.05	69.80	348	572	599
21.8	1.350	13.20	24.20	13.27	69.71	419	530	629
22.0	1.362	13.32	21.85	15.34	69.55	484	479	659
22.5	1.4	13.62	14.8	19.5	68.45	614	325	729
23	1.422	13.92	6.45	21.9	66.82	690	142	766
23.5	1.455	14.22	− 2.5	22.4	64.64	705	− 54.5	835
24	1.485	14.52	−11.2	20.8	62.17	656	−246	862
24.5	1.515	14.82	−18.9	17.9	59.67	549	−416	870

t	αt	βt	$B\cos\beta t$	$A\sin\beta t$	I	$E\sin\beta t$	$F\cos\beta t$	R.P.M.
25	1.545	15.12	-24.9	12.42	57.34	392	-548	858
25.5	1.575	15.42	-28.7	6.35	55.38	193	-631	828
26	1.61	15.72	-29.9	- .27	53.99	- 8.5	-657	788
26.5	1.64	16.02	-28.5	-6.9	53.14	-217	-626	737
27	1.67	16.32	-24.5	-12.9	52.96	-406	-539	683
27.5	1.70	16.62	-18.3	-17.7	53.44	-560	-403	648
28	1.73	16.92	-11.5	-21	54.06	-662	-231	582
28.5	1.76	17.22	- 1.76	-22.4	55.85	-706	- 39	543
29	1.795	17.52	7.16	-21.8	57.57	-686	157	518
29.5	1.825	17.82	15.5	-19.2	59.4	-606	339	506
30	1.855	18.12	22.4	-14.9	61.17	-471	491	508
30.5	1.885	18.42	27.2	- 9.3	62.71	-294	597	523
31	1.915	18.72	29.7	- 2.9	63.95	- 91	653	548
31.5	1.95	19.02	29.5	3.8	64.73	120	648	583
32	1.98	19.35	26.3	10.7	65.1	339	577	625
32.5	2.01	19.65	20.9	16.1	64.95	507	459	664
33	2.04	19.95	13.6	20	64.37	631	298	701
33.5	2.07	20.3	11.4	20.7	63.13	654	250	709
34	2.1	20.58	- 4.76	22.1	62.12	698	-105	756
34.5	2.13	20.85	-12.5	20.4	60.95	643	-274	767
35	2.16	21.18	-20.6	16.2	59.49	513	-453	770
35.5	2.195	21.45	-25.7	11.5	58.41	364	-565	761
36	2.225	21.8	-29.4	4.26	57.29	134	-646	742
36.5	2.26	22.05	-29.9	- 1.32	56.74	- 42	-657	722
37	2.29	22.4	-27.5	- 8.8	56.33	-280	-604	691
37.5	2.32	22.7	-22.8	-14.6	56.33	-460	-499	662

t	αt	βt	$B\cos\beta t$	$A\sin\beta t$	I	$E\sin\beta t$	$F\cos\beta t$	R.P.M.
38	2.35	23	-16	-19	56.46	-600	-351	634
38.5	2.38	23.3	- 7.76	-21.6	57.29	-684	-170	611
39	2.41	23.6	1.14	-22.4	58.9	-702	25	593
39.5	2.44	23.9	9.95	-21.1	59.03	-667	216	581
40	2.475	24.2	17.9	-18	59.99	-568	392	577
40.5	2.505	24.5	24.15	-13.28	60.89	-419	538	580
41	2.54	24.8	28.30	- 7.33	61.65	-231	632	590
41.5	2.57	25.1	29.95	- .672	62.23	- 21	657	606
42	2.60	25.4	28.85	6.00	62.59	+189	643	624
42.5	2.63	25.7	25.25	12.12	62.69	+383	562	645
43	2.66	26.0	19.30	17.12	62.56	542	430	666
43.5	2.69	26.3	11.68	20.65	62.18	652	260	684
44.0	2.72	26.6	3.00	22.3	61.66	705	66	700
44.5	2.75	26.9	- 5.95	22.0	61.02	695	-133	711
45	2.785	27.2	-14.37	19.68	60.33	621	-320	716
45.5	2.81	27.5	-21.50	15.62	59.65	494	-479	717
46	2.845	27.8	-26.75	10.18	59.05	321	-595	711
46.5	2.88	28.1	-29.58	3.82	58.55	121	-657	702
47	2.91	28.4	-29.75	- 2.88	58.22	- 91	-662	689
47.5	2.94	28.7	-27.24	- 9.35	58.06	-294	-606	674
48	2.97	29.0	-22.4	-14.92	58.09	-473	-498	659
48.5	3.00	29.3	-15.49	-19.21	58.27	-607	-345	645
49.0	3.03	29.6	- 7.17	-21.80	58.60	-688	-160	633
49.5	3.065	29.9	1.74	-22.4	58.99	-707	+ 39	623
50	3.09	30.2	10.55	-21.0	59.51	-663	234	617
50.5	3.125	30.5	18.34	-17.75	60.25	-561	408	616

t	αt	βt	$B\cos\beta t$	$A\sin\beta t$	I	$E\sin\beta t$	$F\cos\beta t$	R.P.M.
51	3.155	30.8	24.55	-12.90	60.50	-408	546	617
51.5	3.19	31.1	28.5	- 6.90	60.89	-218	635	622
52	3.22	31.4	29.98	+ .457	61.22	+ 14	658	632
52.5	3.25	31.7	28.45	7.07	61.38	+223	623	643
53	3.28	32.0	24.4	13.05	61.41	412	533	653
53.5	3.31	32.4	15.7	19.1	61.27	604	344	668

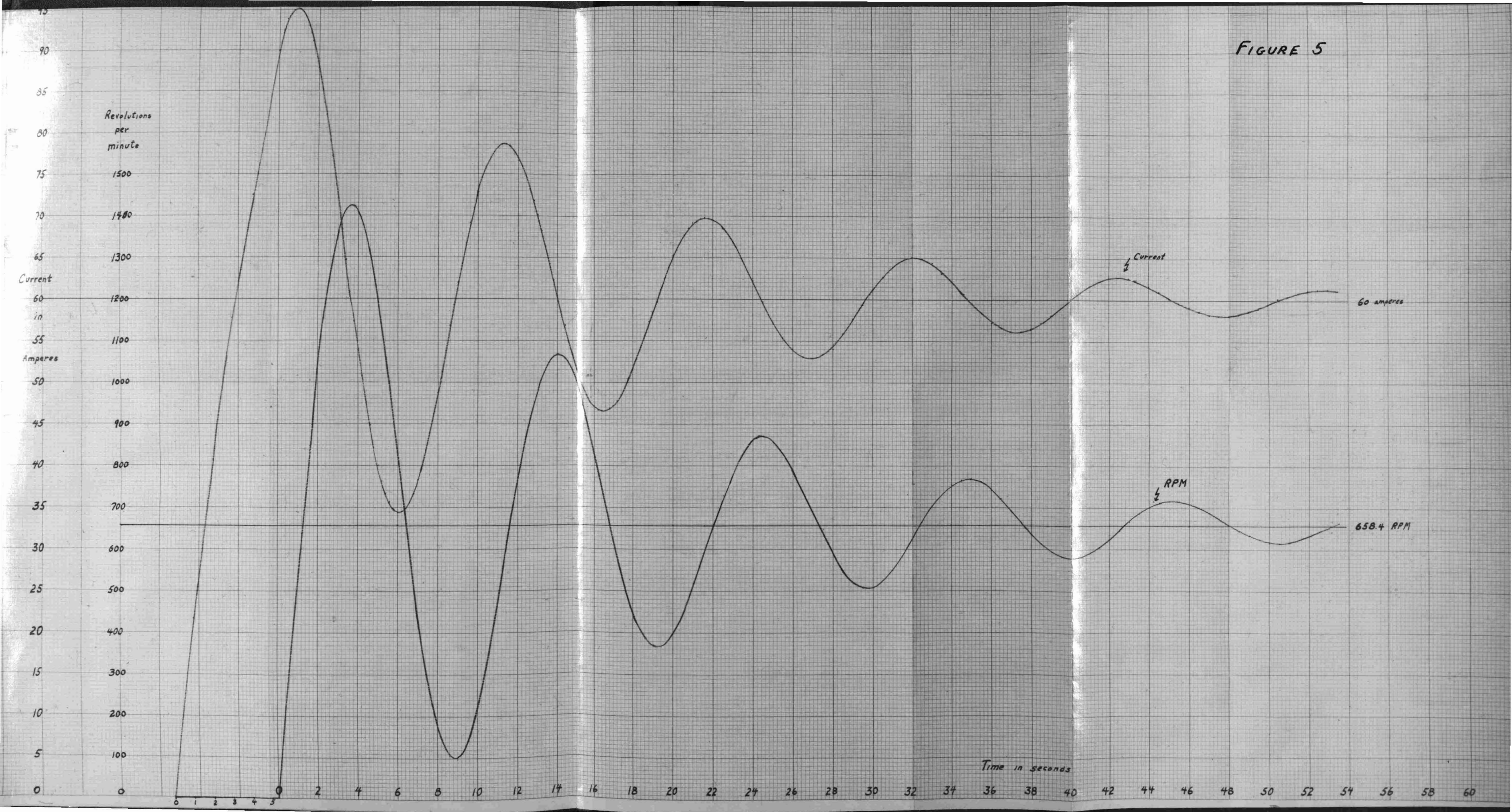
FIGURE 5
Current
in
Amperes
Revolutions
per
minute
Current
RPM
60 amperes
658.4 RPM
Time in seconds
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500
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60
1
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4
5